## STRUCTURE OF FLOW IN THREE-DIMENSIONAL

TURBULENT WAKES
L. N. Ukhanova

UDC 532.526.048. 3

The results are presented for an experimental study of the average flow parameters in threedimensional turbulent wakes which form behind cylinders of finite elongation transverse to the flow and behind a cross-shaped obstacle.

Experimental data on three-dimensional turbulent wakes published earlier have covered the range of Reynolds numbers $\operatorname{Re}=(6.3-70) \cdot 10^{3}[1,2]$. In the present work the results of measurements of the average parameters of the wake behind a cylinder of finite elongation transverse to the flow are presented for a wider range of Reynolds numbers. The range is broadened on the side of higher values up to those close to the critical value (up to $\operatorname{Re}=3.15 \cdot 10^{5}$ ). The effect of the elongation of the cylinder on the average parameters of the three-dimensional wake was studied. Round cylinders having flat ends and elongations $\lambda=5,6.1,8.4,10,13.1$, and 22 were used. In addition, a study was made of the wake behind a cylinder of elongation $\lambda=8.4$ having hemispherical ends. A case of more complex three-dimensional flow was also studied; the wake behind a cross consisting of two mutually perpendicularly crosising cylinders of the same elongation $\lambda=15$, transverse to the flow.

The studies were conducted in wind tunnels of the closed type with an open working section. The cylinders (and the cross) were rigidly fixed with guy wires at the entrance to the working section of the tunnel perpendicular to the impinging stream. The blocking of the tunnel did not exceed $2 \%$ in any of the experiments. The level of turbulence of the impinging stream was less than $0.5 \%$.

The measurements of average velocities were made either with a constant-temperature thermoanemometer having monofilament probes or with Pitot tubes. For measurements in the wakes behind single cylinders the thermoprobe was mounted in the stream so that its filament was parallel to the axis of the cylinder. For measurements in the wake behind the cross the probe was oriented parallel to the bisector of the angle between the cylinders, i,e., at a $45^{\circ}$ angle to the axis of each cylinder. The average velocity profiles in the fixed cross sections were measured both along the axes of symmetry $y$ and $z$ and along lines parallel to them (the directions of the coordinate axes are indicated in Figs. 2 and 4). On the basis of these profiles lines of equal average velocities, isotachs, were constructed at the selected cross sections of the wake. In addition, the transverse dimensions of the wake were estimated from the distributions of average velocity $u(y)$ and $u(z)$ along the axis of symmetry.

As in the studies of flat and axially symmetrical jets and wakes [5], the width of the average velocity profile corresponding to half the velocity defect at the axis was taken as the effective transverse dimension of the three-dimensional wake. Since a three-dimensional wake which forms behind a cylinder of finite elongation transverse to the flow has two planes of symmetry ( $x y$ and $x z$ ) each cross section of such a wake is characterized by two transverse dimensions, $\delta_{y}$ and $\delta_{z}$, determined by the profiles $u(y)$ and $\mathbf{u}(\mathrm{z})$, respectively (see the flow diagram in Fig. 2).

The results of the experiments showed first of all that the nature of the three-dimensional turbulent wake is essentially unchanged in the entire range of Reynolds numbers studied. For the Reynolds numbers $7 \cdot 10^{4} \leq \operatorname{Re} \leq 3.15 \cdot 10^{5}$ the three-dimensional wakes behind cylinders of finite elongation develop in the same way as for $6.3 \cdot 10^{3} \leq \mathrm{Re} \leq 7 \cdot 10^{4}$. In all cases at a certain distance downstream from the cylinder the dimension perpendicular to the axis of the cylinder becomes the larger transverse dimension of the wake.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 25, No. 5, pp. 893-898, November, 1973. Original article submitted October 10, 1972.

[^0]

Fig. 1


Fig. 2

Fig. 1. Average velocity distributions in cross sections of wake behind cylinder ( $\lambda$ $\left.=5, \operatorname{Re}=3.15 \cdot 10^{5}\right)$ : a) average velocity profiles: 1) $\left.\left.\bar{u}(y) ; 2\right) \bar{u}(z) ; b\right)$ isotachs. Solid lines: $\bar{x}=15$; dashed lines: $\bar{x}=5$.
Fig. 2. Variations in $\bar{\delta}_{\mathrm{y}}(\mathrm{x})$ (a) and $\bar{\delta}_{\mathrm{z}}(\mathrm{x})$ (b) for different $\lambda$ : 1) $\lambda=22\left(\mathrm{Re}=1.1 \cdot 10^{4}\right)$; 2) $\lambda=13.1\left(\operatorname{Re}=8.4 \cdot 10^{3}\right)$; 3) $\lambda=10\left(\operatorname{Re}=1.1 \cdot 10^{4}\right)$; 4) $\lambda=6.1\left(\operatorname{Re}=4.2 \cdot 10^{4}\right)$.

The isotachs in two cross sections of the wake behind a cylinder of elongation $\lambda=5$ and at $R e=3.15$ $10^{5}$ are given as an example in Fig. 1. It is seen that the spreading of the wake in the plane of symmetry perpendicular to the axis of the cylinder occurs much faster than in the plane of symmetry passing through its axis. As a result, in cross sections of the wake distant from the axis of the cylinder by five or more diameters the isotachs have close to an elliptical shape with the major axis perpendicular to the axis of the cylinder. The characteristic average velocity profiles $u(y)$ and $u(z)$ in the corresponding cross sections of the wake are also shown in Fig. 1. It should be noted that while at small distances from the cylinder the velocity profiles have a dome-like shape characteristic for flat and axially symmetrical wakes, in more distant sections of the wake in both planes of symmetry the velocity profiles take on a saddle-shaped appearance. In this section of the wake the region of the maximum average velocity defect is shifted away from the longitudinal $x$ axis. Because of the nonuniform variation in velocity across the wake the isotachs corresponding to this section of flow have a shape different from elliptical.

In the entire range of Reynolds numbers examined (from $\operatorname{Re}=6.3 \cdot 10^{3}$ to $R e=3.15 \cdot 10^{5}$ ) the wakes are characterized by a nonuniform distribution of the average velocity of transverse flow. The region of the three-dimensional wake characterized by a saddle-shaped distribution of average velocity in the cross sections is located closer to the cylinder the smaller the elongation $\lambda$. For instance, in the wake behind a cylinder of elongation $\lambda=6.1$ ( $\mathrm{Re}=4.2 \cdot 10^{4}$ ) this section is at a distance $\bar{x} \geq 11$, while in the wake behind a cylinder of elongation $\lambda=22$ ( $\mathrm{Re}=1.1 \cdot 10^{4}$ ) it is at $\bar{x} \geq 70$. It should be mentioned that nonuniformity of the average velocity field of an analogous nature was observed in a study of three-dimensional submerged jets [4].

The clearest dependence of the extent of the sections of flow in which reorganization of the three-dimensional wakes occurs on the elongation of the cylinder can be observed from the variation along the longitudinal coordinate of the effective transverse dimensions of the wake (Fig. 2). For all values of the elongation $\lambda$ examined the transyerse dimension of the wake in the plane of symmetry perpendicular to the axis of the cylinder constantly increases along the entire flow. The transverse dimension of the wake in the plane of symmetry passing through the axis of the cylinder varies little at the start of the flow for small elongations ( $\lambda=5$ and 6.1). For elongations $\lambda \geq 10$ the effective transverse dimension $\delta_{\mathrm{z}}$ decreases markedly at the start of the flow, i.e., narrowing of the wake occurs in the xz plane of symmetry. As a result, the two transverse dimensions become equal at some distance from the axis of the cylinder determined by the elongation $\lambda$. The size of $\delta_{z}$ continues to decrease somewhat, but starting with a certain cross section of the wake further downstream both effective transverse dimensions increase but at different rates.


Fig. 3


Fig. 4

Fig. 3. Attenuation of three-dimensional wakes behind cylinder with flat ends (a) and behind cylinder with hemispherical ends (b): 1) $\bar{u}_{1 \mathrm{~m}}$; 2) $\varepsilon, \%$; 3) $\bar{\delta}_{y}$; 4) $\bar{\delta}_{z}$.
Fig. 4. Isotachs in cross sections of wake behind cross: a) $\bar{x}=5$; b) $\bar{x}$ $=30: 1$ ) $\overline{\mathrm{u}}=0.5$; 2) 0.75 ; 3) 0.9 .

Obviously, in the cross section of the wake in which the two effective transverse dimensions are equal the isotach corresponding to half the velocity defect at the axis is close to circular. Upstream from this cross section it has an elliptical shape with the major axis parallel to the axis of the cylinder. In the cross sections of the wake lying downstream from the section where $\delta_{y}=\delta_{z}$ the isotach again takes a shape close to elliptical but with the major axis perpendicular to the axis of the cylinder. It is clear that the isotachs corresponding to other average velocities undergo analogous variations. However, the lower the velocity corresponding to the isotachs, the closer to the cylinder do they acquire a circular shape. For example, for the wake behind a cylinder of elongation $\lambda=22$ the distances along the flow from the axis of the cylinder to the cross sections where the transverse dimensions of the wake are equal in the two planes of symmetry are approximately 62,54 , and 46 diameters for one fourth, one half, and three fourths of the average velocity defect, respectively. Thus, in observing the reorganization of the three-dimensional wake from oval in cross section with a major axis parallel to the axis of the cylinder to oval with a major axis perpendicular to the axis of the cylinder it is impossible to isolate an intermediate region where the flow in the wake would be close to axially symmetrical.

As noted earlier, the extent of the region of a three-dimensional wake in which its reorganization occurs depends essentially on the elongation of the cylinder. If the distance $\bar{x}_{0}=x_{0} / d$ from the axis of the cylinder to the cross section in which $\delta_{y}=\delta_{z}$ is taken as the characteristic length then from the results of the studies conducted one can obtain the following equation

$$
\bar{x}_{0}=f(\lambda)=0.2 \lambda^{1.75}
$$

(for the range of Reynolds numbers $\mathrm{Re}=(0.63-3.13) \cdot 10^{4}$ and the range of variation of cylinder elongation from 5 to 22).

The elongation of the cylinder not only determines the extent of the separate characteristic regions of the three-dimensional flow but also affects the nature of the attenuation of the wake. The attenuation of three-dimensional wakes was observed from the distribution along the flow of the dimensionless average velocity defect. It was found that the three-dimensional wake dies out faster, the less elongated the cylinder.

The properties of the three-dimensional wake mentioned above are characteristic both for the wakes behind cylinders with flat ends and for the wakes behind cylinders with hemispherical ends. As an illustration, the experimental data on the parameters of the wakes behind two cylinders of the same elongation $\lambda$ $=8.4$ but with different ends are presented in Fig. 3. It is seen that even at small distances from the cylinders the two wakes develop almost identically. The variations in the effective transverse dimensions
along the longitudinal coordinate and the patterns of attenuation of the wake both in average velocity and in its longitudinal pulsation coincide in the two cases.

It must be noted that the three-dimensional wakes along the flow behind bodies of more complicated configuration are deformed in an analogous way. For example, in studying the flow behind a cross from the variation in the shape of the isotachs in different cross sections of the wake it was found that in this case there is a tendency toward more rapid spreading of the wake in the direction of the bisector of the angle between the axes of the cylinders forming the cross (Fig. 4). At large enough distances from the cross in the region of flow where the velocity defects are small the isotachs in the cross sections of the wake become almost circular, i.e., along the stream in the three-dimensional wake behind a cross the flow is reorganized into axially symmetrical flow.

In analyzing the experimental data obtained on the parameters of three-dimensional wakes one can divide the wake behind a cylinder of finite elongation transverse to the flow into characteristic regions, as has been done, for example, in the study of three-dimensional submerged jets [3].

A zone of return flow forms in the immediate vicinity behind the cylinder. Its extent along the flow and along the span of the cylinder depends on the elongation of the cylinder. For example, while for a cylinder of infinite span the length of the zone of return currents is about 1.5 diameters [6], for a cylinder of elongation $\lambda=5$ this length reaches three diameters in the direction of the x axis $\left[\right.$ at $\left.R e=(7-31.5) \cdot 10^{\frac{1}{4}}\right]$ 。

Behind the zone of return currents extends a section of the wake in which the spreading of the wake in the plane of symmetry perpendicular to the axis of the cylinder and its contraction in the plane of symmetry passing through the axis of the cylinder occur simultaneously. This region of the flow can be arbitrarily bounded by the cross section starting with which both transverse dimensions of the wake ( $\delta_{\mathrm{y}}$ and $\delta_{\mathrm{z}}$ ) increase along the stream. As the measurements showed (see Fig. 2), the extent of this region also depends on the elongation of the cylinder.

For large cylinder elongations one can distinguish in this section of the three-dimensional wake a region of flow in the vicinity of the $x$ axis which develops after the fashion of a flat close wake [6]. Here the average velocity remains constant along the $z$ axis, the velocity profiles $u(y)$ are similar, the transverse component of the pulsation velocity exceeds the longitudinal component as in a flat close wake, and a vortex structure similar to a Kármán street is observed.

The following third section is characterized by an increase along the flow of both transverse dimensions of the wake. Here $\delta_{y}>\delta_{z}$, i.e., the larger transverse dimension in this section is perpendicular to the axis of the cylinder. The variation in the average velocity across the wake occurs unevenly here. The region of maximum velocity defect is shifted from the longitudinal axis of flow. The average velocity profiles $u(y)$ and $u(z)$ have a saddle-shaped form.

It can be assumed that as in the attenuation of three-dimensional submerged jets [3], farther downstream the three-dimensional wake is transformed into axially symmetrical flow. However, it has not yet been possible to conduct reliable measurements in this region in connection with the strong attenuation of three-dimensional wakes along the flow.

## NOTATION

$\lambda=l / \mathrm{d} ;$
$l$
d
$\frac{x}{x}=x / d ;$
$\frac{y}{y}=y / d ;$
$\frac{\mathrm{z}}{\mathrm{Z}}=\mathrm{z} / \mathrm{d}$;
${ }_{\mathrm{u}}^{\mathrm{u}}{ }^{\delta}$
$\bar{u}=\mathbf{u} / u_{\delta} ;$
$\frac{u_{m}}{\bar{u}_{1 m}}=\left(u_{\delta}-u_{m}\right) / u_{\delta}$
is the cylinder length;
is the diameter;
is the longitudinal coordinate;
is the transverse coordinate perpendicular to axis of cylinder;
is the transverse coordinate parallel to axis of cylinder;
is the velocity of impinging stream;
is the longitudinal component of average velocity;
is the velocity at axis of wake;
is the velocity defect at axis of wake;

| $\vec{\delta}_{\mathrm{y}}=\delta_{\mathrm{y}} / \mathrm{d}$ and $\vec{\delta}_{\mathrm{z}}=\delta_{\mathrm{z}} / \mathrm{d}$. | are the effective transverse dimensions of wake; <br> $\mathbf{x}_{0}=\mathrm{x}_{0} / \mathrm{d}$ <br> Re |
| :--- | :--- |
| is the distance from axis of cylinder to cross section where $\delta_{\mathrm{y}}=\delta_{\mathrm{z}} ;$ |  |
| $\boldsymbol{v}$ | is the Reynolds number; |
| $\varepsilon$ | is the coefficient of kinematic viscosity; |
| is the intensity of turbulence, $\%$. |  |

## LITERATURE CITED

1. Y. Kuo and L. V. Baldwin, J. Fluid Mech., 27, No. 2, 353-361 (1967).
2. L. N. Ukhanova, Uchenye Zapiski Tsentr. Aero-Gidrodinam. In-ta, 2, No. 6, 93-97 (1971).
3. P. M. Sforza, M. N. Steiger, and N. Trentacoste, AIAA J., 4, 800-806 (1966).
4. P. M. Sforza and N. Trentacoste, ibid., 5, 885-890 (1967).
5. A. S. Ginevskii, Theory of Turbulent Jets and Wakes [in Russian], Mashinostroenie, Moscow (1969).
6. L. N. Ukhanova, Industrial Aerodynamics [in Russian], Oborongiz (1966), Part 27.

[^0]:    © 1975 Plenum Publishing Corporation, 227 West 17th Street, New York, N.Y. 10011 . No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, microfilming, recording or otherwise, without written permission of the publisher. A copy of this article is available from the publisher for $\$ 15.00$.

